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RELIABILITY AND PROFIT ANALYSIS OF A STANDBY UNIT SYSTEM WITH CORRELATED LIFE TIME IN AN INDUSTRY

MOHIT KUMAR KAKKAR AND JASDEV BHATTI

ABSTRACT. The main objective of this work is to present a reliability analysis of a cold standby unit system with variations in production and demand, As we know that Demand of a product in the market influences the production of that product and also the business of the company or an organization. we have assumed that only one repair person is available in the whole system. Failure and Repair times of each unit are supposed to be dependent or correlated. By utilizing regenerative point technique, number of reliability attributes are obtained which are very vital and helpful to the framework planners, mechanical supervisors and industrial managers. Graphical behaviors of mean time to system failure and profit function have also been studied.

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KEYWORDS AND PHRASES. Profit Analysis, MTSF, Availability, Busy Period.

1. INTRODUCTION

A two- unit cold standby system with one repairperson has been one of the classical models in the literature of reliability theory. Several studies including Mokaddis *et al.* (2008);Kadyan *et al.*(2004)and Gupta *et al.*(1996) have analyzed a two unit model with different repair policies and different type of maintenance. In all of these studies they have assumed that Demand is fixed but in real world this is not so. Such type of situation may be arised in any industry where production depends on the demand .Keeping the above real situation in view, we, in the present paper analyze a two similar unit cold standby system model in which workload on a unit is higher when Demand is greater than production.The proposed approach has been applied to the working two unit cold standby system of a paper mill situated in northern part of India.Keeping the above situation in view in this work we investigates the reliability analysis of a two unit cold standby system assuming the possibility of variation in demand and production and repair times of each unit as correlated random variables having their joint distribution as bivariate exponential.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS

The following description of system is as follows:

- The system consists of two identical units,one is operative and other is in cold standby state.

- Initially,system starts its operation from state S_o in which one of the unit is in operative mode.
- Work load on the system varies for different demand and production.

3. TRANSITION DIAGRAM AND NOTATIONS

For defining the states of the system we assume the following symbols:

A_o :	Unit A is in active mode
A_s :	Unit A is in stand by/retaining mode
A_{fr} :	Unit A is in Fizzled state(mode)
A_{fw} :	Unit A is in waiting for repairman
u_1 :	Constant rate of increment of production over Demand
u_2 :	Constant rate of increment of Demand over Production
X_i :	Random variables representing the failure times of A and B unit respectively
Y_i :	Random variables representing the repair times of A and B unit respectively
$f_i(x,y)$:	Joint pdf of (x_i, y_i) $=\alpha_i\beta_i(1-r_i)e^{-\alpha_i x-\beta_i y}I_0 2\sqrt{\alpha_i\beta_i r_i xy}$
$k_i(\frac{Y}{X})$:	conditional pdf of y_i given $X_i = x$: is given by $\beta_i e^{-\alpha_i r_i x-\beta_i y}I_0 2\sqrt{\alpha_i\beta_i r_i xy}$
$g_i(.)$:	marginal pdf of $X_i = \alpha_i(1-r_i)e^{-\alpha_i(1-r_i)x}$
$h_i(.)$:	marginal pdf of $y_i = \beta_i(1-r_i)e^{-\beta_i(1-r_i)y}$
$\bigoplus$ ::	symbol of convolution $A(t)\bigoplus B(t) = \int_0^t A(t-u)B(u)du$
$\bigotimes$ ::	symbol of stieltjes convolution$A(t)\bigotimes B(t) = \int_0^t A(t-u)dB(u)$
a :	Probability of repair when both units are in failure mode(Demand $<$ production)
b :	Probability of repair when both units are in failure mode(Demand $<$ production)
m :	Probability of repair when both units are in failure mode(Demand $>$ production)
n :	Probability of repair when both units are in failure mode(Demand $>$ production)
p :	Probability of repair when one unit is in failure mode(Demand $<$ production)
q :	Probability of repair when one unit is in failure mode(Demand $<$ production)
q_ij :	pdf of transition time from regenerative states Si to Sj.
Q_ij :	cdf of transition time from regenerative states Si to Sj.
μ_i :	Mean sojourn time in state Si.

3.1. Transition Probabilities.

The steady state transition probability can be as follows

(1) $$p_{01} = \frac{u_1}{u_1+\alpha_1(1-r_1)}$$

(2) $$p_{02} = \frac{\alpha_1(1-r_1)}{u_1+\alpha_1(1-r_1)}$$

(3) $$p_{13} = \frac{\alpha_1(1-r_1)}{u_2+\alpha_1(1-r_1)}$$

(4) $$p_{10} = \frac{p}{p+q+\alpha_1(1-r_1)}$$

(5) $$p_{25} = \frac{\alpha_1(1-r_1)}{\beta_1(1-r_1)+\alpha_1(1-r_1)}$$

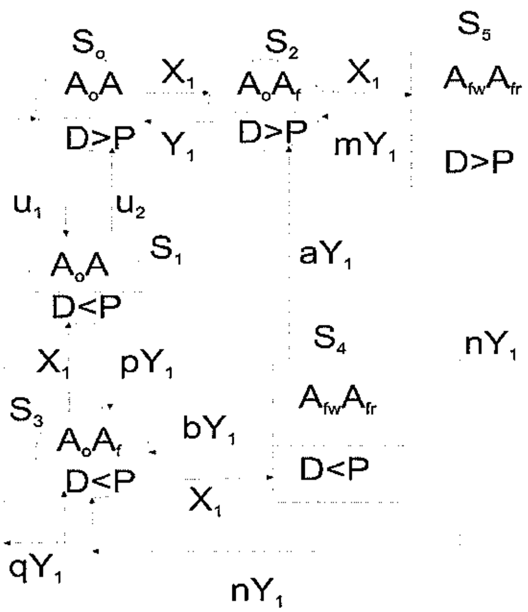

FIGURE 1. Transition Diagram

(6) $$p_{20} = \frac{\beta_1(1-r_1)}{\beta_1(1-r_1)+\alpha_1(1-r_1)}$$

(7) $$p_{30} = \frac{q}{\alpha_1(1-r_1)+q+p}$$

(8) $$p_{53} = \frac{n}{m+n}$$

(9) $$p_{52} = \frac{m}{m+n}$$

(10) $$p_{42} = \frac{a}{a+b}$$

(11) $$p_{43} = \frac{b}{a+b}$$

4. ANALYSIS OF CHARACTERISTICS

4.1. **Mean Time to System Failure.** To determine the MTSF of the system, we regard the failed state of the system as absorbing state, by probabilistic arguments, we get

(12) $$\phi_0(t) = Q_{01}(t) \bigotimes \phi_1(t) + Q_{02}(t) \bigotimes \phi_2(t)$$

(13) $$\phi_1(t) = Q_{13}(t) \bigotimes \phi_3(t) + Q_{10}(t) \bigotimes \phi_0(t)$$

(14) $$\phi_2(t) = Q_{20}(t) \bigotimes \phi_0(t) + Q_{25}(t)$$

$$\phi_3(t) = Q_{30}(t) \bigotimes \phi_0(t) + Q_{31}(t) \bigotimes \phi_1(t) + Q_{34}(t) \tag{15}$$

Taking Laplace stieltjes transforms of these relations and solving for $\phi_0^{**}(s)$,

$$\phi_0^{**}(s) = \frac{N(S)}{D(S)} \tag{16}$$

where

$$N = \mu_0(p_{01} + 1 - p_{13}p_{31}) + (\mu_1 + \mu_3)(1 - p_{25}p_{31}) + \mu_2(1 - p_{02}) \tag{17}$$

$$D = \mu_0(1 - p_{13}p_{31}) + \mu_1 p_{01} + \mu_2 p_{02}(1 - p_{13}p_{31}p_{01}) + \mu_3 p_{01}p_{13} \tag{18}$$

4.2. **Availability Analysis.** Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at $t = 0$. Using the arguments of the theory of a regenerative process the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations when (D>P)

$$A_0(t) = M_0(t) + q_{01}(t) \bigoplus A_1(t) + q_{02}(t) \bigoplus A_2(t) \tag{19}$$

$$A_1(t) = q_{10}(t) \bigoplus A_0(t) + q_{13}(t) \bigoplus A_3(t) \tag{20}$$

$$A_2(t) = M_2(t) + q_{20}(t) \bigoplus A_0(t) + q_{22.5}(t) \bigoplus A_2(t) + q_{23.5}(t) \bigoplus A_3(t) \tag{21}$$

$$A_3(t) = q_{30}(t) \bigoplus A_0(t) + q_{31}(t) \bigoplus A_1(t) + q_{33.4}(t) \bigoplus A_3(t) + q_{32.4}(t) \bigoplus A_2(t) \tag{22}$$

$$A_0^*(s) = \frac{Nd_1(S)}{D_1(S)} \tag{23}$$

The steady state availability is

$$A_0^d = \lim_{s \to 0}(sA_0^*(s)) = \frac{Nd_1}{D_1} \tag{24}$$

$$\begin{aligned} Nd_1 &= \mu_0[p_{01}p_{02}(p_{13}p_{30}p_{21.8} + p_{39}p_{21.8} - p_{24}) + p_{30.6}p_{90}p_{13}p_{39}(1 - p_{410}p_{24}) \\ &\quad - p_{90}p_{13}p_{21.8}p_{30.6} + p_{24}p_{13}p_{30.6}p_{40.5}p_{24} + p_{24}p_{90}p_{39} \\ &\quad - p_{13}p_{31} + (1 - p_{12.7}p_{21.6})] \end{aligned} \tag{25}$$

$$\begin{aligned} D_1 &= \mu_0(p_{01}p_{13} + p_{33.4}p_{31}p_{22.5} - p_{25}p_{31}p_{13}p_{20}) + \mu_1(p_{20}p_{10} + p_{42}p_{25}p_{01}) \\ &\quad + \mu_2 p_{22.5}(1 - p_{13}p_{33.4} - p_{01}p_{13}) + \mu_3 p_{25}(p_{13}p_{22.5} + p_{13}p_{01}p_{23.5} + p_{31}p_{13}p_{23.5}) \end{aligned}$$

when (D<P)

$$A_0(t) = q_{01}(t) \bigoplus A_1(t) + q_{02}(t) \bigoplus A_2(t) \tag{26}$$

$$A_1(t) = M_1(t) + q_{10}(t) \bigoplus A_0(t) + q_{13}(t) \bigoplus A_3(t) \tag{27}$$

$$A_2(t) = q_{20}(t) \bigoplus A_0(t) + q_{22.5}(t) \bigoplus A_2(t) + q_{23.5}(t) \bigoplus A_3(t) \tag{28}$$

$$A_3(t) = M_3(t) + q_{30}(t) \bigoplus A_0(t) + q_{31}(t) \bigoplus A_1(t) + q_{33.4}(t) \bigoplus A_3(t) + q_{32.4}(t) \bigoplus A_2(t) \tag{29}$$

Now taking Laplace transform of these equations and solving them for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{Np_1(S)}{D_1(S)} \tag{30}$$

The steady state availability is

$$A_0^p = \lim_{s\to 0}(sA_0^*(s)) = \frac{Np_1}{D_1} \tag{31}$$

(32) $Np_1 \;= \mu_1(p_{01}p_{02}(p_{13}p_{30}p_{21.8} - p_{24})) + p_{30.6}p_{90}p_{13}p_{39}(1 - p_{12.7}p_{21.6})$

$$\begin{aligned} D_1 \;&= \mu_0(p_{01}p_{13} + p_{33.4}p_{31}p_{22.5} - p_{25}p_{31}p_{13}p_{20}) + \mu_1(p_{20}p_{10} + p_{42}p_{25}p_{01}) \\ &+\mu_2 p_{22.5}(1 - p_{13}p_{33.4} - p_{01}p_{13}) + \mu_3 p_{25}(p_{13}p_{22.5} + p_{13}p_{01}p_{23.5} + p_{31}p_{13}p_{23.5}) \end{aligned}$$

4.3. **Busy period analysis of the repairman.** Let $B_i(t)$ be the probability that the repairman is busy at instant t, given that the system entered regenerative state I at $t = 0$. By probabilistic arguments we have the following recursive relations for $B_i(t)$.

$$B_0(t) = q_{01}(t) \bigoplus B_1(t) + q_{02}(t) \bigoplus B_2(t) \tag{33}$$

$$B_1(t) = q_{10}(t) \bigoplus B_0(t) + q_{13}(t) \bigoplus B_3(t) \tag{34}$$

(35) $B_2(t) = W_2(t) + q_{20}(t) \bigoplus B_0(t) + q_{22.5}(t) \bigoplus B_2(t) + q_{23.5}(t) \bigoplus B_3(t)$

(36)
$B_3(t) = W_3(t) + q_{30}(t) \bigoplus B_0(t) + q_{31}(t) \bigoplus B_1(t) + q_{33.4}(t) \bigoplus B_3(t) + q_{32.4}(t) \bigoplus B_2(t)$

Now taking Laplace transform of these equations and solving them for $B_0^*(s)$, we get

$$B_0^*(s) = \frac{N_2(S)}{D_1(S)} \tag{37}$$

The steady state availability is

$$B_0 = \lim_{s\to 0}(sB_0^*(s)) = \frac{N_2}{D_1} \tag{38}$$

where

$$\begin{aligned} N_2 \;&= \mu_1[p_{01} - p_{25}p_{01} + p_{13} + p_{20}p_{22.5}] + \mu_2[p_{01}p_{13}(1 + p_{43}) + p_{31}p_{01}p_{32.4} \\ &+p_{25}p_{02}(1 - p_{13}p_{33.4})] + \mu_3[p_{02}p_{42}p_{20} - p_{01}p_{13}(p_{30}) \\ &-p_{01}p_{13}p_{25}] \end{aligned}$$

D_1 has already been evaluated.

4.4. **Expected number of visits by the repairman.** We defined as the expected number of visits by the repairman in $(0, t]$,
given that the system initially starts from regenerative state S_i, By probabilistic arguments we have the following recursive relations for $V_i(t)$

$$V_0(t) = Q_{01}(t) \bigoplus (1 + V_1(t)) + Q_{02}(t) \bigoplus (1 + V_2(t)) \tag{39}$$

$$V_1(t) = Q_{10}(t) \bigoplus V_0(t) + Q_{13}(t) \bigoplus V_3(t) \tag{40}$$

$$V_2(t) = Q_{20}(t) \bigoplus V_0(t) + Q_{22.5}(t) \bigoplus V_2(t) + Q_{23.5}(t) \bigoplus V_3(t) \tag{41}$$

(42)
$$V_3(t) = Q_{30}(t) \bigoplus V_0(t) + Q_{31}(t) \bigoplus V_1(t) + Q_{33.4}(t) \bigoplus V_3(t) + Q_{32.4}(t) \bigoplus V_2(t)$$

Taking laplace stieltjes transform of the equations of expected number of visits And solving them for $V_0^{**}(s)$, we get

$$V_0^{**}(s) = \frac{N_3(S)}{D_1(S)} \tag{43}$$

in steady state

$$V_0 = \lim_{s \to 0}(s v_0^{**}(s)) = \frac{N_3}{D_1} \tag{44}$$

$$N_3 \quad = p_{02}(1 - p_{13}p_{31}) + p_{01}p_{13}p_{25} \tag{45}$$

D_1 has already been evaluated.

5. PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$Profit(P) = C_0 A_0^d - C_1 A_0^p - C_2 B_0 - C_3 V_0 \tag{46}$$

where
C_0 =Revenue per unit uptime of the system when production is less than Demand
C_1 =Revenue per unit uptime of the system when production is greater than Demand
C_2 =Cost per unit time for which repairman is busy in repair
C_3 =Cost per visit of the repairman

6. CONCLUSION

For a more clear view of the system characteristics w.r.t. the various parameters involved, we plot curves for MTSF and Profit Function in figure-2, figure-3, figure-4 and figure-5 w.r.t the repair rate and failure rate parameters of unit A for three different values of correlation coefficient ($r_{11} = 0.25, r_{12} = 0.50, r_{13} = 0.75$), between X_1 and Y_1 while the other parameters are kept fixed as
$\alpha_2 = 0.005, \beta_1 = 0.03, \beta_2 = 0.02, \theta = 0.001, r_2 = 0.50$
$C_0 = 700, C_1 = 500, C_2 = 30$

figure-2 is depicted that MTSF increases as repair rate increases irrespective of other parameters. This graph also showing that for the fixed value of repair rate, Mean time to system failure is higher for higher values of correlation coefficient(r), so here we can see that the high value of (r) between failure and repair tends to increase the expected life time of the system. Also figure-3 reveals the variation in profit with respect to the repair rate and we can see that profit is also moving upper side of the graph as repair rate increases. Also for the fixed value of repair rate the profit is higher for high correlation (r). It can be interpreted from Figure-4 that as the failure rate is moving right hand side of the axis, Mean time to system failure is coming downward direction, this conclude that reliability of the system is also diminishing as the fizzled rate is increasing. The observations drawn from the figure-5 is more interesting as for $r_{11} = 0.25$ the profit of the system is +ve , =0, or -ve according as the failure rate is <, =, or > 0.02745 , as for $r_{12} = 0.50$ the profit of the system is +ve , =0, or -ve according as the failure rate is <, =, or > 0.0367 and as for $r_{13} = 0.75$ the profit of the system is +ve , =0, or -ve according as the failure rate is < ,=, or > 0.0466 and we can see that as failure rate of unit A is increasing profit of the system is diminishing rapidly. At this stage we can conclude that threshold limit for numerous failure rates, repair rates can be achieved which helps in deciding the maximum and minimum acceptable values of rates and even of costs so that the system is profitable. Hence the upper limit of the failure rate can be obtained so that the system can give the positive profit.

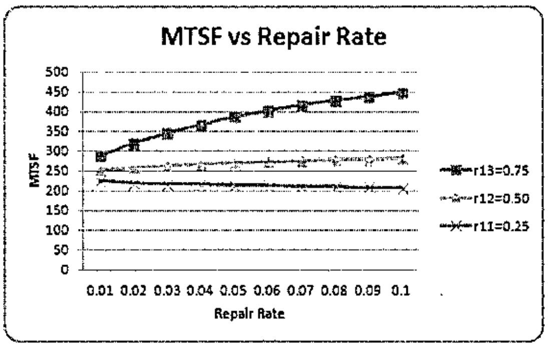

FIGURE 2. MTSF vs Repair Rate

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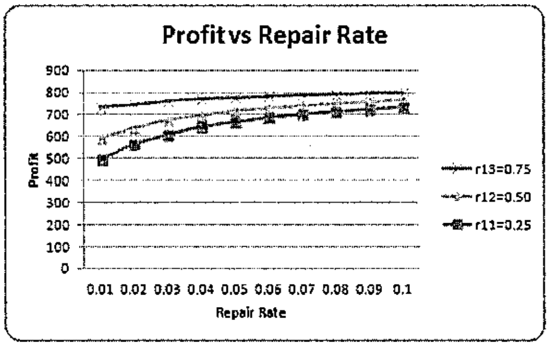

FIGURE 3. Profit vs Repair Rate

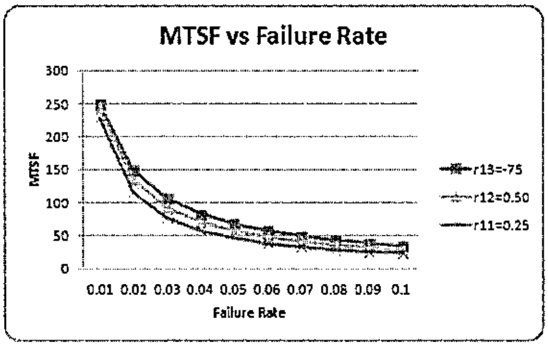

FIGURE 4. MTSF vs Failure Rate

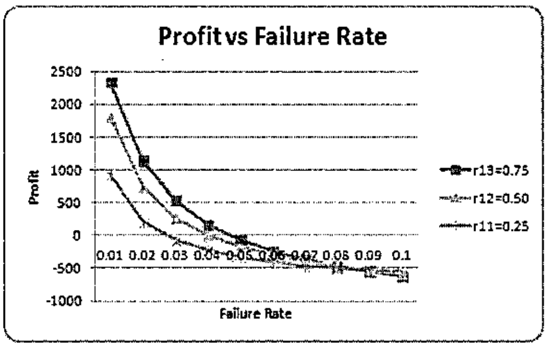

FIGURE 5. Profit vs Failure Rate

ASSOCIATE PROFESSOR,CHITKARA UNIVERSITY, RAJPURA, PUNJAB, INDIA
E-mail address: mohitkakkar2001@gmail.com

ASSITANT PROFESSOR,CHITKARA UNIVERSITY, RAJPURA, PUNJAB, INDIA
E-mail address: jasdevbhatti1983@gmail.com